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CLIMATOLOGY OF MONTHLY PRECIPITATION PATTERNS IN THE WESTERN UNITED STATES, 1931–1966

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ABSTRACT

The eigenvector or "empirical orthogonal function" approach is used to determine the dominant precipitation anomaly patterns for the western United States for each month during the last 36 yr. In all months there is enough intercorrelation among monthly precipitation amounts in different parts of the region that at least 45 percent of the total variance can be explained by only three eigenvectors. Usually the most important pattern is one with a single large region of anomalous precipitation, centered in southern California, Arizona, or Nevada in winter and in Washington, Idaho, or Montana in summer. Also important in all months is a pattern with anomalies of opposite sign in the Pacific Northwest and the Arizona-New Mexico-Texas area.

1. INTRODUCTION

In 1963 the U.S. Weather Bureau [20] published average values of precipitation and temperature for all State climatic divisions and for each month from January 1931 through December 1960. These data provide an excellent point-of-departure for describing the general climatology of the region for the period considered. By averaging values within State climatic divisions, much of the intrastate random variability is eliminated and what emerges is a fairly smooth and consistent picture of the spatial distributions of temperature and precipitation.

This paper might be considered an extension of work done principally by Namias [13], Klein [9], Gilman [5, 6], Stidd [17], and their associates in the Extended Forecast Division of the U.S. Weather Bureau. However, the emphasis here is not on forecasting but, rather, on the climatology of the monthly precipitation distribution for the western United States. An attempt will be made to delineate, for each month, certain precipitation patterns that have tended to recur in several different years between 1931 and 1966. These patterns will be related, at least partially, to anomalies in the air flow in the middle troposphere.

Use is made of the eigenvector or "empirical orthogonal function" approach introduced into meteorology by Lorenz [11] in 1956. In the present context the basic method consists of taking all 35 or 36 of the precipitation anomaly maps for a given month and condensing them to a small number of patterns or eigenvectors which explain most of the total variance of the field. Unlike the original anomaly maps, the eigenvectors are uncorrelated with one another. Except for being objectively determined, they are similar to "weather types." Each eigenvector has associated with it a coefficient or amplitude whose magnitude and sign vary from year to year, depending on how closely the eigenvector resembles the particular precipitation pattern.

Eigenvector analysis, under a variety of names, has been used in many different problems in meteorology during the past decade by White et al. [22], Aubert et al. [3], Grimmer [7], Steiner [16], Mateer [12], Wark and Fleming [21], Stidd [18], Christensen and Bryson [4], Alishouse et al. [2], and Kutzbach [10].

The data used in this study were taken primarily from [20] for the period from 1931 through 1960 and from [19] for the various States for the period from 1961 through April 1966. The 50 climatic divisions selected are listed in table 1 and located in figure 1. Upper air data at 700 mb. were taken from *Monthly Weather Review* before 1950 and from *Climatological Data, National Summary* thereafter.

In the following sections, the eigenvector method will be described briefly and then applied to the problem at hand.

2. THEORY

Most of the material to be presented in this section is taken from Lorenz [11] and Sellers [15]. We can start

TABLE 1.—*The 50 climatic divisions used in this study*

Area	Division	State
1	West Olympic Coastal	Washington
2	Northeastern	Washington
3	Central Basin	Washington
4	Willamette Valley	Oregon
5	South Central	Oregon
6	North Coast Drainage	California
7	Northeast Interior Basins	California
8	Central Coast Drainage	California
9	San Joaquin Drainage	California
10	Southeast Desert Basins	California
11	South Coast Drainage	California
12	Central Mountains	Idaho
13	Central Plains	Idaho
14	Northeastern	Nevada
15	South Central	Nevada
16	Extreme Southern	Nevada
17	Western	Utah
18	Northern Mountains	Utah
19	Southeastern	Utah
20	Northeastern	Arizona
21	East Central	Arizona
22	Southeastern	Arizona
23	Western	Montana
24	North Central	Montana
25	Southwestern	Montana
26	South Central	Montana
27	Big Horn and Wind River Drainage	Wyoming
28	Platte Drainage	Wyoming
29	Colorado Drainage Basin	Colorado
30	Platte Drainage Basin	Colorado
31	Arkansas Drainage Basin	Colorado
32	Northwestern Plateau	New Mexico
33	Central Highlands	New Mexico
34	Northeastern Plains	New Mexico
35	Southern Desert	New Mexico
36	Northeastern	North Dakota
37	West Central	North Dakota
38	North Central	South Dakota
39	Black Hills	South Dakota
40	Southeastern	South Dakota
41	North Central	Nebraska
42	South Central	Nebraska
43	West Central	Kansas
44	North Central	Oklahoma
45	High Plains	Texas
46	Low Rolling Plains	Texas
47	North Central	Texas
48	Trans Pecos	Texas
49	Edwards Plateau	Texas
50	Southern	Texas

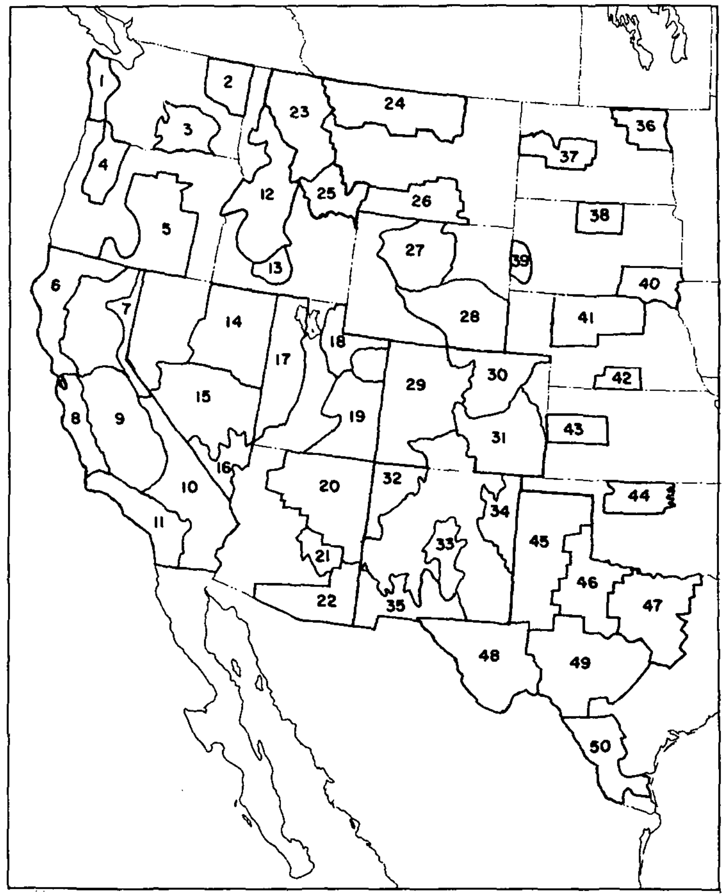


FIGURE 1.—Locations of the 50-State climatic divisions selected for use in this study.

off by letting $\mathbf{P}$ be an $n\times m$ matrix of, say, average January precipitation for m climatic divisions and a series of n years. The element p_{ij} in the ith row and jth column of this matrix represents the average January precipitation in the jth division during the ith year. In this study, m equals 50 and n equals 36 (January through April) or 35 (May through December). The precipitation values for each month have been standardized by subtracting out the division mean and dividing by the standard deviation. This is a step of convenience rather than of necessity, since, in this case, the $m\times m$ matrix $n^{-1}\mathbf{P}'\mathbf{P}=n^{-1}\mathbf{A}$, where $\mathbf{P}'$ is the transpose of $\mathbf{P}$, is a correlation matrix with ones along the principal diagonal.

The next step is to let

$$\mathbf{P}=\mathbf{QF} \text{ or } p_{ij}(x,y,t)=\sum_{k=1}^{m} q_{ik}(t)\, f_{kj}(x,y), \tag{1}$$

where $\mathbf{F}$ is specified to be an orthonormal $m\times m$ matrix for which

$$\mathbf{FF}'=\mathbf{I}, \tag{2}$$

where $\mathbf{I}$ is an identity matrix, with zeros everywhere except along the principal diagonal, whose elements are all unity. Because $\mathbf{A}$ is a symmetric matrix, we may further specify that

$$\mathbf{FAF}'=\mathbf{Q}'\mathbf{Q}=\mathbf{D} \tag{3}$$

be a diagonal matrix with non-zero values d_k only along the principal diagonal. The matrices $\mathbf{F}$ and $\mathbf{D}$ are, respectively, the eigenvector and eigenvalue matrices of the matrix $\mathbf{A}$. The kth row vector $\mathbf{F}_k$ of the matrix $\mathbf{F}$ is the kth eigenvector and varies only in space, corresponding to the kth empirical orthogonal function of Lorenz [11]. Similarly the kth column vector $\mathbf{Q}_k$ of the matrix $\mathbf{Q}$ is the coefficient or amplitude vector of the kth eigenvector and varies only in time. From equation (3) it follows that the $\mathbf{Q}_k$'s are uncorrelated and also that the sum of the squares of the elements q_{ik} of $\mathbf{Q}_k$ equals d_k, i.e.,

$$\sum_{i=1}^{n} q_{ik}^2=d_k. \tag{4}$$

The matrices **D** and **F** are usually determined by either the power method of Aitken [1] or the diagonalization method of Jacobi [8]. Both are discussed by Ralston and Wilf [14, chapters 7 and 18] and by Sellers [15].

It is rarely necessary to determine all of the m eigenvectors. Often only two or three will explain most of the variance of the field being specified. When the p_{ij} are expressed either as deviations from the mean or as standardized variables, this variance is given by

$$\frac{1}{n}\sum_{j=1}^{m}\left(\sum_{i=1}^{n}p_{ij}^2\right)=\sum_{j=1}^{m}\overline{p_j^2}, \tag{5}$$

which reduces to m, the number of climatic divisions when standardized variables are used. But, from equations (1) and (2),

$$\mathbf{PP'}=\mathbf{QQ'} \text{ or } \sum_{j=1}^{m}p_{ij}^2=\sum_{k=1}^{m}q_{ik}^2. \tag{6}$$

Thus, summing both sides of this equation over the n years, reversing the order of summation, and using equations (4) and (5) gives

$$\sum_{j=1}^{m}\overline{p_j^2}=\frac{1}{n}\sum_{k=1}^{m}d_k. \tag{7}$$

For standardized variables the sum of the m eigenvalues equals nm and the fraction of the total variance explained by the kth eigenvector equals d_k/nm.

Two derived variables will be used in the following analysis. The first is r_{ik}^2, the square of the correlation coefficient between $\mathbf{P}_i$ and $\mathbf{F}_k$ for the ith month and the kth eigenvector. This tells us what fraction of the space variance of precipitation in the ith month is explained by the kth eigenvector. Since the correlation coefficient is a test only of similarity of patterns and not of absolute magnitudes, a second derived variable, v_{ik}^2, was used. This is defined by

$$v_{ik}^2=q_{ik}^2\Big/\sum_{j=1}^{m}p_{ij}^2 \tag{8}$$

and, from equation (6), gives the fraction of the sum of squares of the p_{ij} for the ith month accounted for by the kth eigenvector. r_{ik}^2 and v_{ik}^2 need not be correlated, although in practice they usually are, as shown in figure 2 for January and July and for the first and second eigenvectors. r_{ik} and v_{ik} will be equal when the space mean values of p_{ij} and f_{kj} are zero. This is more likely to be true for the higher order eigenvectors than for the first eigenvector, for which both p_{ij} and f_{kj} usually have the same sign over practically the whole region.

The latter feature results from a predominance of positive correlations of monthly precipitation amounts among the 50 climatic divisions. The percentage of the 1,225 correlations computed for each month that were positive ranged from 58.4 percent in August to 82.4 percent in October. The largest positive correlation was 0.985 between divisions 10 and 11 in southern California in March. These two regions yielded the highest positive correlation in all months except May through August and October. The largest negative correlation was −0.567 between divisions 4 and 46 and divisions 22 and 42 in June. Most of the high negative correlations were between Texas and Washington and Oregon. For all months combined, 2,189 (15 percent) of the 14,700 correlations were greater than or equal to 0.5; only 14 were less than or equal to −0.5; and 9,455 (64 percent) had an absolute magnitude of less than 0.30.

3. RESULTS

In table 2 is given for each month the accumulated variance explained by the first m eigenvectors, with m ranging from 1 to 10. Averaged for all months, the first three eigenvectors explain slightly more than half of the total variance. These are shown, by months, in figures 3 through 8. Generally, the first eigenvector explains about 25 percent of the total variance, the second 18 percent, and the third 11 percent.

In the figures, isolines of f_{kj} equal to 0.2, 0.1, −0.1, and −0.2 have been drawn. Although it is impossible to relate these directly to precipitation anomalies, more often than not the following association can be made:

f_{kj}	*precipitation anomaly*
greater than 0.2	much above normal
0.1 to 0.2	above normal
0.1 to −0.1	near normal
−0.1 to −0.2	below normal
less than −0.2	much below normal

The years in which the observed precipitation anomaly pattern most closely resembled each of the given eigenvector fields and the respective values of v^2 and r^2 are listed in the figures. The resemblance may be either positive or negative. If positive, the anomalies are as given in the table above; if negative, the signs are all reversed and regions of much above normal precipitation become regions of much below normal precipitation. This is one of the advantages of eigenvector analysis over conventional weather typing, in that it allows the signs of the anomalies to go either way. For the most part, the years listed are those for which the geometric mean of v^2 and r^2 exceeded 0.25.

Anomalies in the precipitation field should reflect anomalies in the circulation pattern aloft. Therefore it seemed appropriate to superimpose on each of these figures the pattern of 700-mb. height difference between the years with the highest positive and the highest negative correlations with the eigenvector field. The years used are indicated by asterisks. Isolines of height difference at 20-m. intervals are drawn in the figures; only the zero isoline is labeled. Data for this analysis were avail-

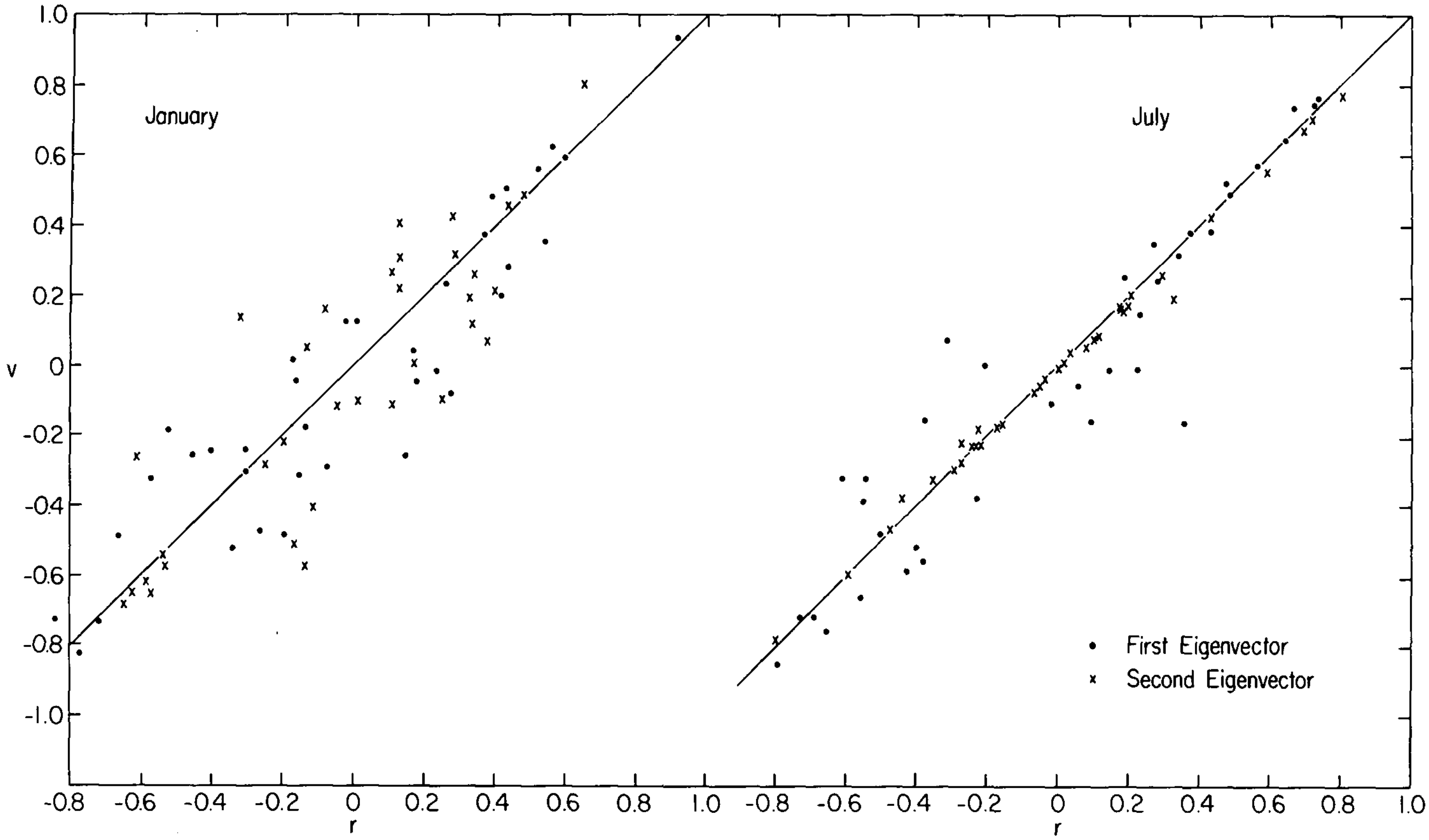


FIGURE 2.—Relationship between r_{ik} and v_{ik} for January and July and for the first and second eigenvectors.

TABLE 2.—*Percentage of the total variance explained by the first m eigenvectors*

Month	*m*									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
January	25.8	47.5	58.1	65.7	71.5	76.3	79.6	82.6	85.2	87.5
February	26.6	42.3	54.7	62.9	68.4	73.4	76.8	79.9	82.7	85.2
March	31.2	46.0	56.4	63.3	68.2	71.9	75.6	78.5	81.3	83.9
April	27.9	48.1	57.5	64.5	70.4	74.5	78.1	81.2	83.7	85.8
May	25.2	47.1	58.4	64.8	69.7	73.9	77.3	80.3	82.8	85.1
June	23.7	40.4	50.8	59.0	65.1	70.5	74.5	78.0	81.3	83.9
July	17.1	32.5	45.6	54.1	61.7	67.9	72.0	75.6	78.8	81.3
August	21.3	37.4	47.8	56.7	63.1	68.9	73.6	77.2	80.1	82.7
September	26.1	42.5	55.0	63.0	70.4	74.6	78.2	81.2	84.1	86.3
October	30.2	48.0	57.8	65.3	70.5	74.3	78.0	81.2	83.7	86.0
November	24.6	46.9	58.4	66.1	71.7	76.6	80.5	83.8	86.1	88.4
December	27.9	47.9	56.6	63.9	70.2	74.5	78.4	81.7	84.8	87.0
Average	25.6	43.9	54.8	62.4	68.4	73.1	76.9	80.1	82.9	85.3

able only for the period starting in January 1939. These patterns appear to be fairly realistic, especially in winter, and indicate more or less what one might expect. However, they should be taken only as guides to the circulation aloft until a more quantitative correlation between precipitation and air flow is available.

Rather than going into any great detail, in the following brief mention will be made of the more interesting features of the three most important eigenvector fields, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, respectively for each month. Before starting, however, it should be pointed out that the explained variances given in table 2 and figures 3 to 8 are averages for the 50-division area. The percentage of the total variance of precipitation in the jth division explained by the kth eigenvector is given by

$$\text{explained variance} = 100(d_k/n)f_{kj}^2.$$

This quantity varies over the grid. The patterns shown in figures 3 to 8 are those which are dominant for the region as a whole and not necessarily for particular divisions. For example, most of the variance of precipitation in southern Texas in January is explained by eigenvectors 2, 4, and 6, not 1, 2, and 3. In only eight of the 40 divisions are the latter the dominant eigenvectors.

The total variance explained by the first three eigenvectors is generally highest in the western part of the grid and in winter. Average maxima range from about 70 to 90 percent. The lowest values are usually found along the Olympic coast of Washington, in northeastern North Dakota, and in southern Texas; minima normally lie between 10 and 30 percent.

Considering figures 3 to 8 as a whole, there are four basic anomaly patterns that seem to occur repeatedly. Usually the most important is one with a single large

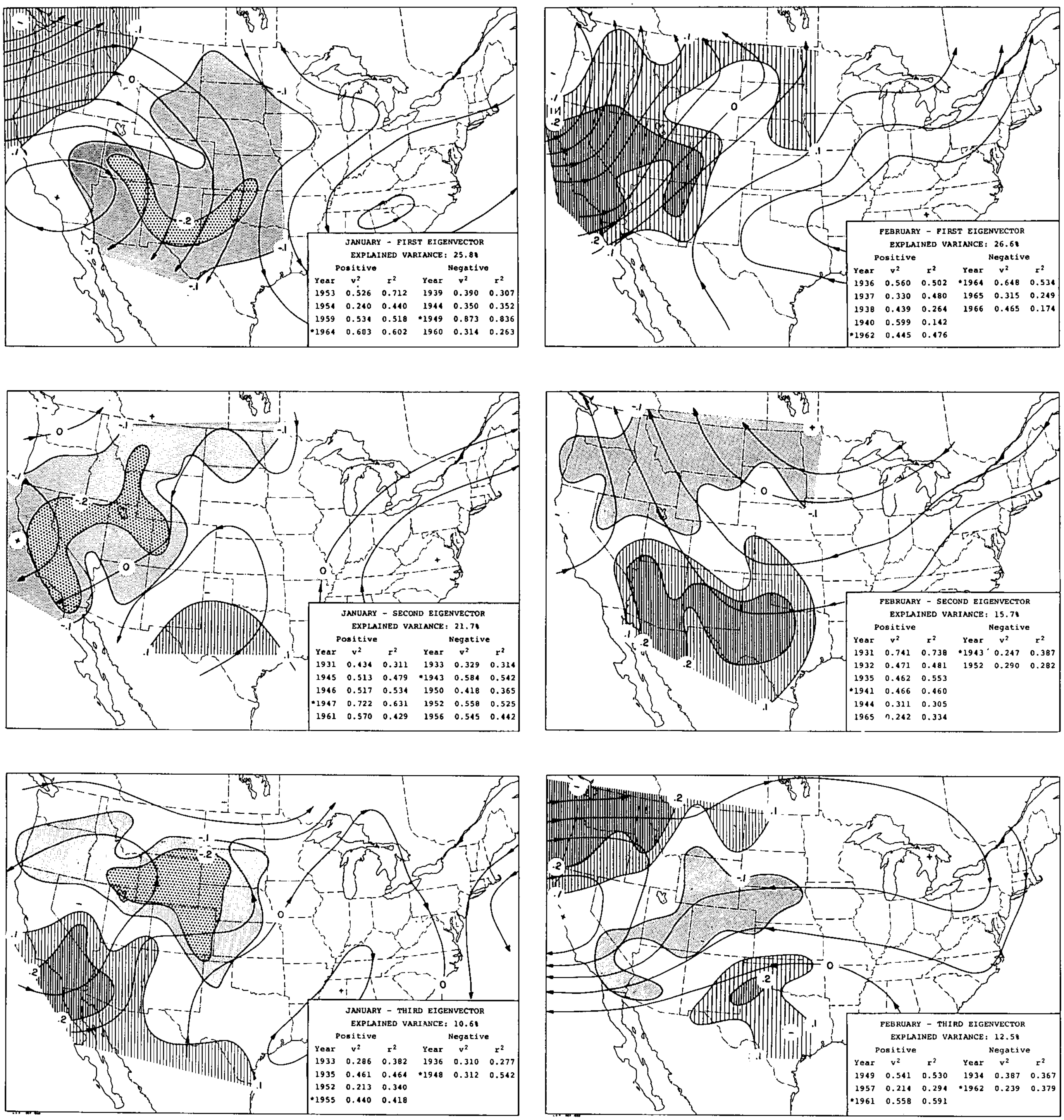


FIGURE 3.—The first three eigenvectors for January and February. The insert table gives the percentage of the total variance explained by each eigenvector and the years in which the observed precipitation anomaly pattern and the eigenvector were most similar, either positively or negatively. The solid lines with arrow heads give the 700-mb. height difference between the years with the highest positive and negative correlations with the eigenvector field. See the text for further details.

region of anomalous precipitation, centered in southern California, Arizona, or Nevada in winter and in Washington, Idaho, or Montana in summer. The dominance of this pattern can be related to the overwhelming abundance of positive correlations in the **A** matrix. From the height difference fields it appears that the pattern may be associated mainly with east-west shifts of the major middle latitude pressure centers. In general, for all eigenvectors it will be noticed that regions of heavy precipitation occur where the anomaly flow aloft

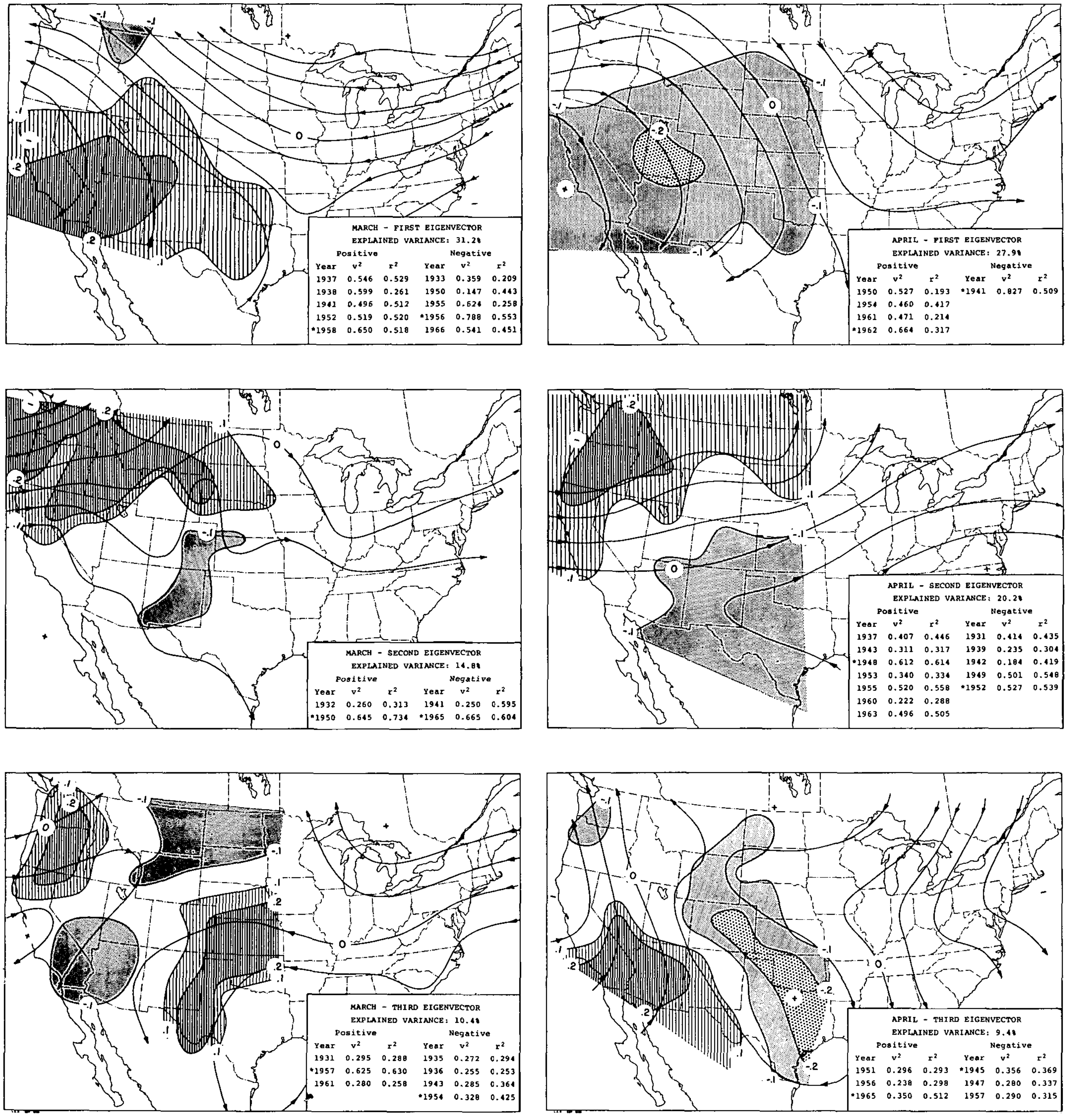


FIGURE 4.—Same as figure 3, except for March and April.

is cyclonic and from either the Pacific Ocean or the Gulf of Mexico.

In each month there is one eigenvector with precipitation anomalies of opposite signs in the Pacific Northwest and the Arizona-New Mexico-Texas area. This opposition is probably associated with shifts in the locations of the storm track. When the track is north of its normal position, as it was during much of 1943, 1948, 1953, and 1956, heavy rain falls in the Northwest, while drought occurs in the Southwest. When the storm track is to the south, as it was in 1941, 1960, and 1965, the Southwest is wet and the Northwest is dry.

The third basic pattern, which is best developed in the late winter and spring, is characterized by anomalies of

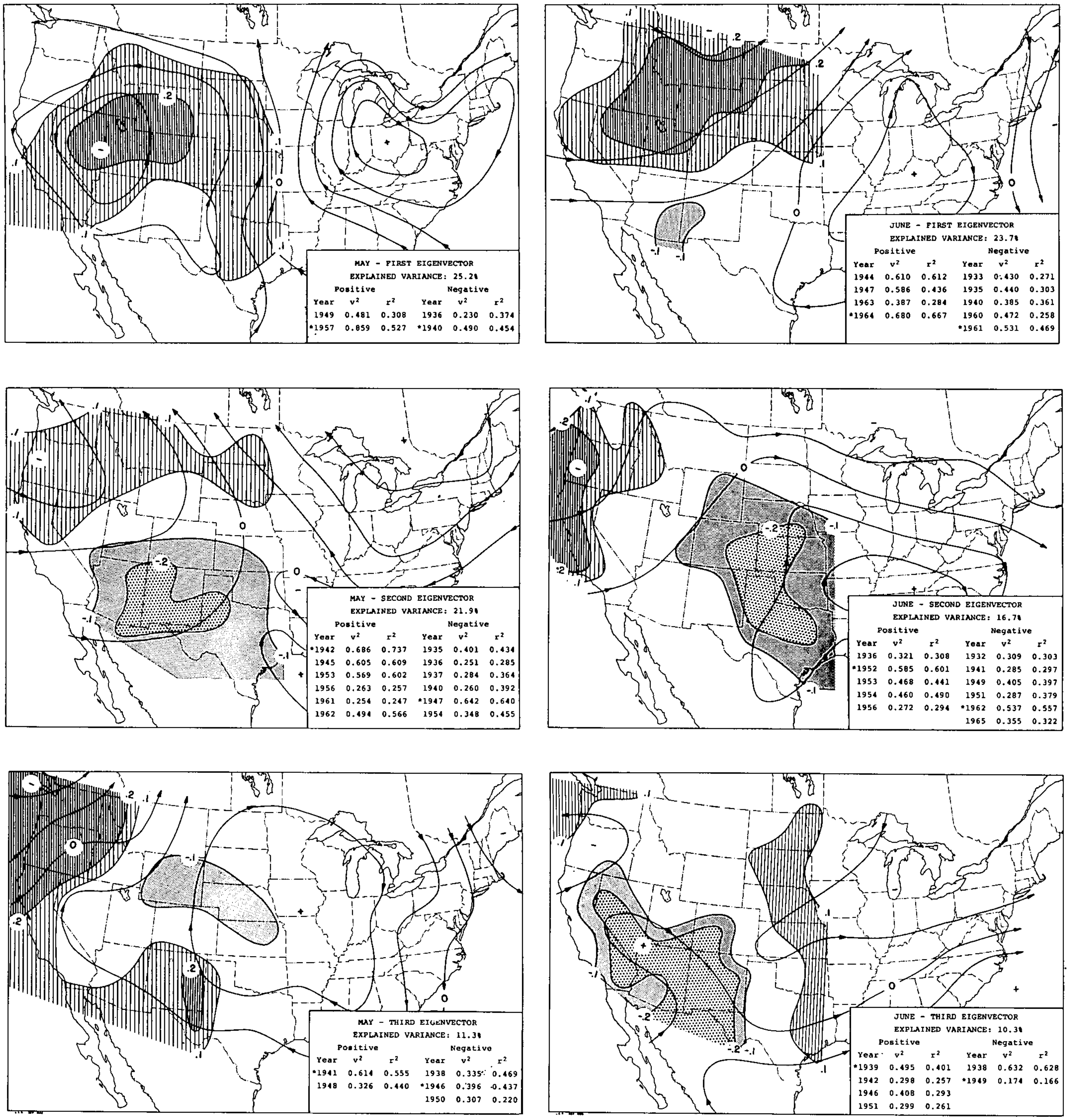


FIGURE 5.—Same as figure 3, except for May and June.

one sign in the northwest and southeast portions of the area and of the opposite sign in the southwest and northeast portions. The third eigenvector for March is the best example of this.

The fourth pattern features anomalies of opposite sign in the southwestern and northeastern portions of the area. It may be at least partially induced by the Rocky Mountains and is most prominent in the fall and early winter.

There are a few features of the individual patterns that should be mentioned. In some cases a particular eigenvector is dominant for 3 or 4 yr. in succession. This is true, for example, of F_2 in January from 1945 to 1947, F_1 in February from 1936 to 1940 and again from 1964

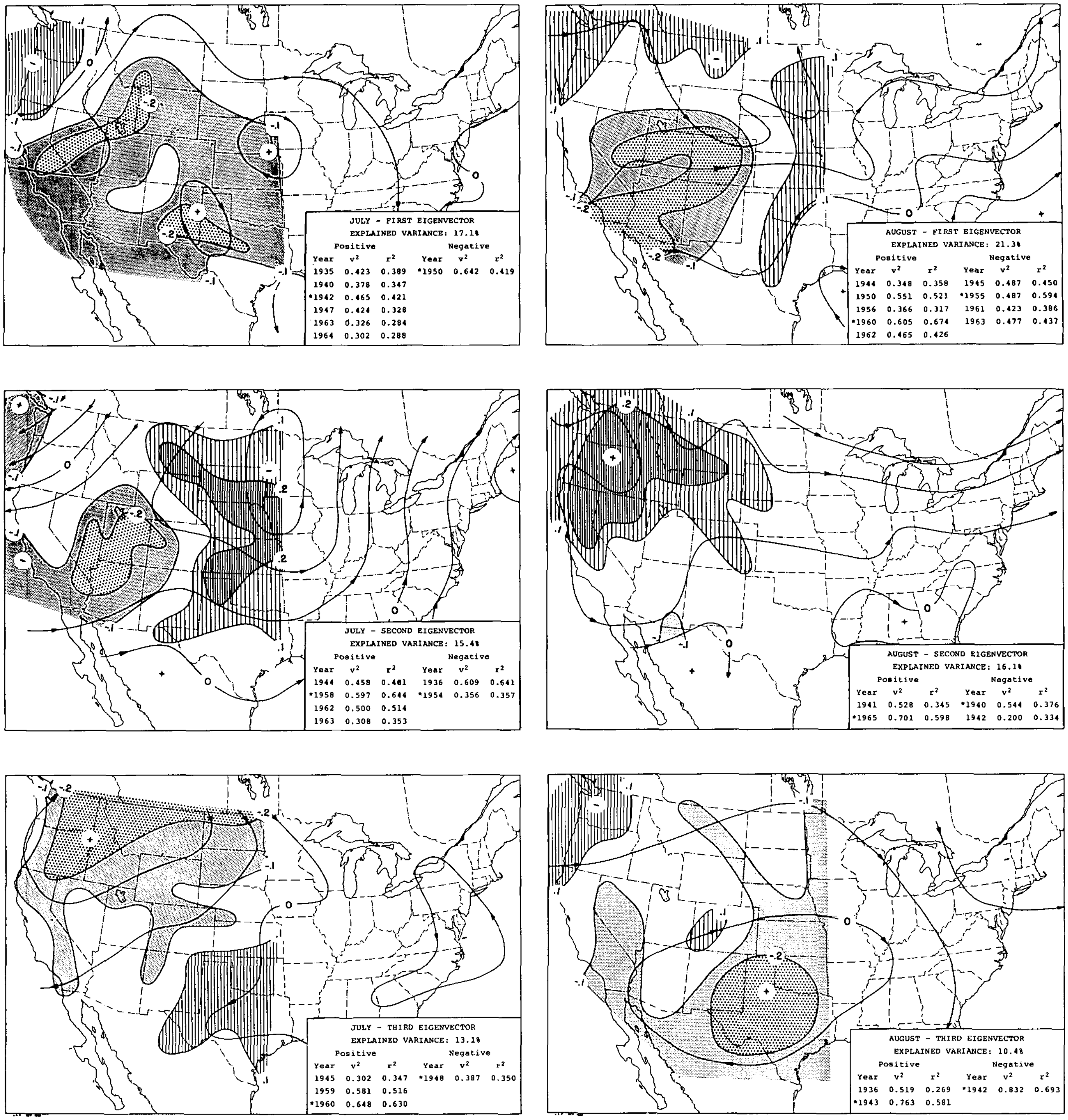


FIGURE 6.—Same as figure 3, except for July and August.

to 1966, and $\mathbf{F}_2$ in June from 1952 to 1956. The same type of behavior is also apparent in August. However, here the sign of the anomaly reverses from one year to the next. Each of the three eigenvectors for August is dominated by a single region of anomalous precipitation. The first, associated with $\mathbf{F}_1$, is located in the southwestern portion of the region; the second, with $\mathbf{F}_2$, is in the Northwest, and the third, with $\mathbf{F}_3$, is in Texas. In the case of $\mathbf{F}_1$, the Southwest was dry in 1944 and wet in 1945, wet in 1955 and dry in 1956, and dry in 1960 and 1962 and wet in 1961 and 1963. Similarly, the Northwest was dry in 1940, wet in 1941 and dry again in 1942, and Texas was wet in 1942 and dry in 1943.

Although they are interesting and perhaps suggestive of systematic variations, these trends occur so infrequently and so irregularly that it is doubtful whether

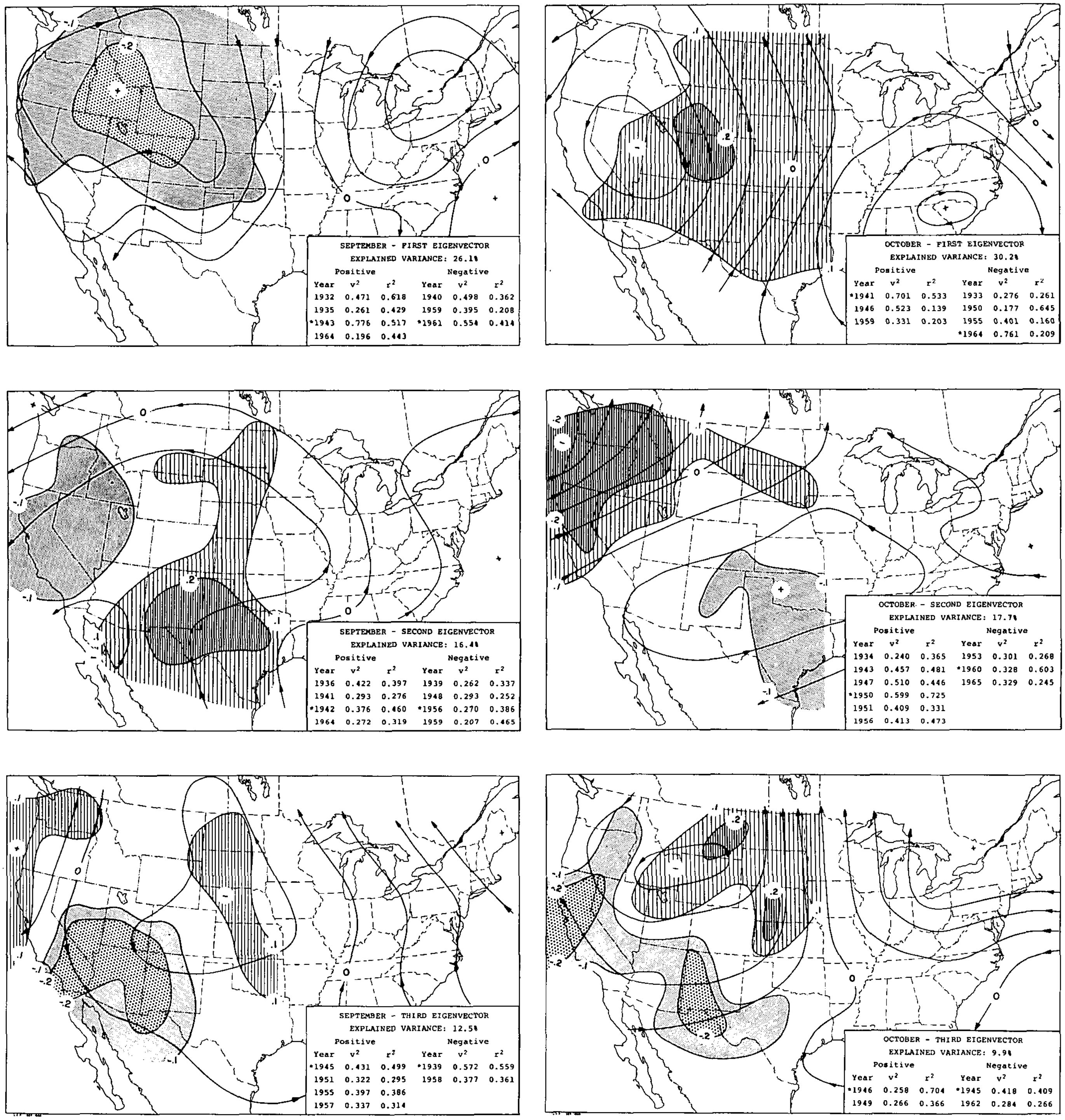
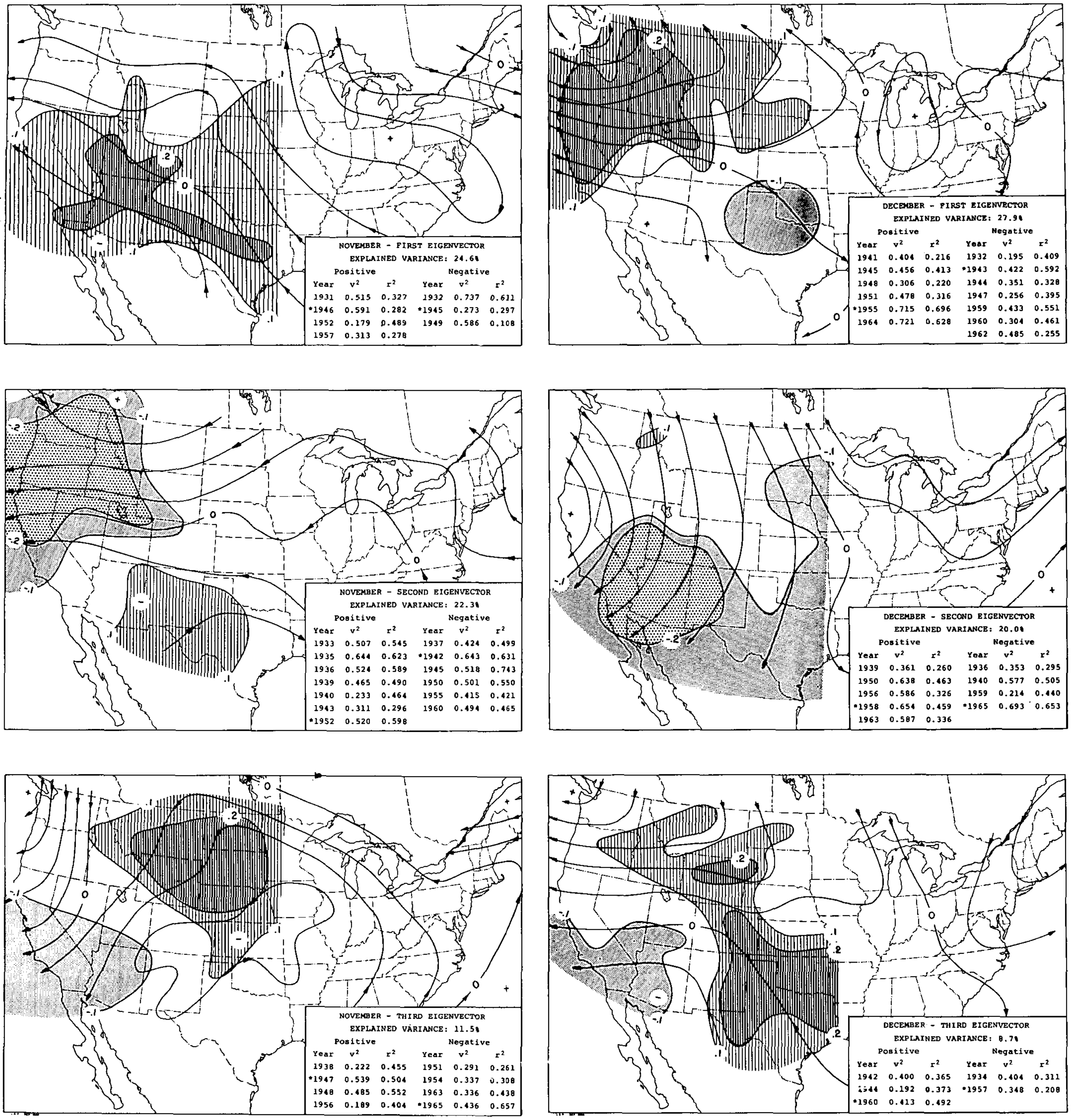


FIGURE 7.—Same as figure 3, except for September and October.

they have any forecasting value. Actually, it is not even obvious that the eigenvectors that have been most important during the last 30 yr. will remain so during the next 30 yr. Conceivably, completely different regimes will prevail.

Most of the summer rain that falls in the Southwest is usually attributed to an intensification and westward extension of the Bermuda High, with a strong flow of unstable warm, moist air into the region from the Gulf of Mexico. This seems to be borne out by the second eigenvector for July and the first one for August. Both of these patterns indicate that enhanced precipitation in the Southwest is favored by a strong ridge over the Great Plains. At the same time, however, local 700-mb. heights may be near or even below normal. When the ridge develops to the south, over Texas, summer drought is



NOVEMBER - FIRST EIGENVECTOR
EXPLAINED VARIANCE: 24.6%

Positive Year	v^2	r^2	Negative Year	v^2	r^2
1931	0.515	0.327	1932	0.737	0.611
*1946	0.591	0.282	*1945	0.273	0.297
1952	0.179	0.489	1949	0.586	0.108
1957	0.313	0.278			

DECEMBER - FIRST EIGENVECTOR
EXPLAINED VARIANCE: 27.9%

Positive Year	v^2	r^2	Negative Year	v^2	r^2
1941	0.404	0.216	1932	0.195	0.409
1945	0.456	0.413	*1943	0.422	0.592
1948	0.306	0.220	1944	0.351	0.328
1951	0.478	0.316	1947	0.256	0.395
*1955	0.715	0.696	1959	0.433	0.551
1964	0.721	0.628	1960	0.304	0.461
			1962	0.485	0.255

NOVEMBER - SECOND EIGENVECTOR
EXPLAINED VARIANCE: 22.3%

Positive Year	v^2	r^2	Negative Year	v^2	r^2
1933	0.507	0.545	1937	0.424	0.499
1935	0.644	0.623	*1942	0.643	0.631
1936	0.524	0.589	1945	0.518	0.743
1939	0.465	0.490	1950	0.501	0.550
1940	0.233	0.464	1955	0.415	0.421
1943	0.311	0.296	1960	0.494	0.465
*1952	0.520	0.598			

DECEMBER - SECOND EIGENVECTOR
EXPLAINED VARIANCE: 20.0%

Positive Year	v^2	r^2	Negative Year	v^2	r^2
1939	0.361	0.260	1936	0.353	0.295
1950	0.638	0.463	1940	0.577	0.505
1956	0.586	0.326	1959	0.214	0.440
*1958	0.654	0.459	*1965	0.693	0.653
1963	0.587	0.336			

NOVEMBER - THIRD EIGENVECTOR
EXPLAINED VARIANCE: 11.5%

Positive Year	v^2	r^2	Negative Year	v^2	r^2
1938	0.222	0.455	1951	0.291	0.261
*1947	0.539	0.504	1954	0.337	0.308
1948	0.485	0.552	1963	0.336	0.438
1956	0.189	0.404	*1965	0.436	0.657

DECEMBER - THIRD EIGENVECTOR
EXPLAINED VARIANCE: 8.7%

Positive Year	v^2	r^2	Negative Year	v^2	r^2
1942	0.400	0.365	1934	0.404	0.311
1944	0.192	0.373	*1957	0.348	0.208
*1960	0.413	0.492			

FIGURE 8.—Same as figure 3, except for November and December.

likely over most of the Southwest, and especially in Texas.

4. CONCLUSIONS

This has been a preliminary study, the main purpose of which was to see if there is enough intercorrelation among monthly precipitation amounts in different parts of the western United States to permit delineating certain area-wide "typical" patterns. The conclusion is that there is, with an average of slightly more than half of the total variance of precipitation being explained in each month by three such patterns.

The next step is to determine how closely the precipitation anomaly field is related to the wind and temperature fields aloft. The most direct approach, and one which is independent of what has been presented here, would be to use stepwise regression, with the precipitation within each division as the predictand and height and temperature data at a large number of grid points at 700 or 500 mb. as the possible predictors. A second, and perhaps more satisfying approach, would also use stepwise regression. However, here the coefficients of the most important eigenvectors of the precipitation anomaly field would be predicted from the coefficients of the eigenvectors of the height and temperature anomaly fields. The first approach involves point correlations, the second area correlations. For this reason, the latter might be expected to yield more stable predictions and be fairly insensitive to small data errors. However, it would require a great deal more time and probably yield less impressive results, at least when applied to the dependent data.

The use of monthly means in a climatological study such as this is usually considered quite appropriate. However, a shorter, 5-day or weekly, averaging period might yield better and more satisfying results. For one thing the sample size would be greatly increased. Also, there would be less chance of averaging through several different weather regimes. On the other hand, data collection becomes a problem. Average precipitation data for the various climatic divisions are available only on a monthly basis. Either the shorter period values would all have to be computed or single station data used. The latter possibility is not very appealing.

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